Multi-target tracking using Probability Hypothesis Density (PHD) filters for whistle contour detection

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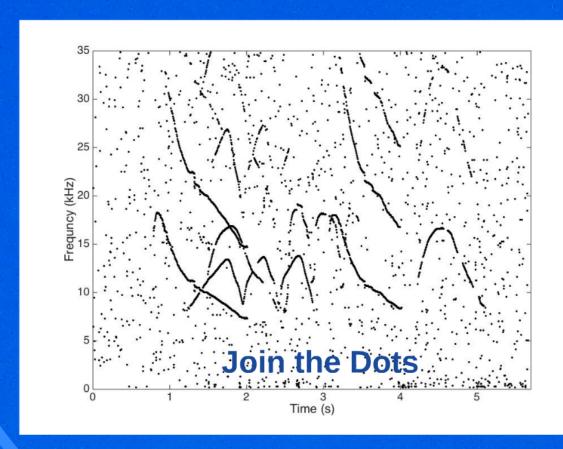
Institute of Sound and Vibration Research



Overview

- Target tracking
 - Single target tracking
 - Multi-target tracking
- PHD filter GM-PHD filter
- GM-PHD application to synthetic and real data
- Challenges
- Conclusions

Problem description



Target tracking

Target: anything whose state is of interest

State: contains all information about the system under observation

at a particular time

Measurement: output from a sensor (target + clutter)

Target tracking = the process of estimating a target's state from noisy measurements and linking together the detections from one source

2 models required

System (dynamic) model

$$x_k = Fx_{k-1} + Qv_{k-1}$$

Describes transformation of the state (x_k) with time

Measurement (observation) model

$$x_k = Fx_{k-1} + Qv_{k-1}$$
 $z_k = Hx_k + Rw_k$

Describes how what we measure (z_k) relates to the state (x_k)

State space model for dolphin whistles

State

$$x_k = \begin{bmatrix} f & \alpha \end{bmatrix}^t \text{ Current whistle frequency} \\ \text{ Current whistle chirp rate} \\$$

System model
$$x_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_{k-1} + v_{k-1}$$
 System noise

Measurement
$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + w_k$$
 model We only measure Measurement noise frequency

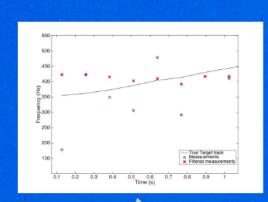
Single target tracking

- One target present, all measurements generated by target
- Kalman filter

$$x_k = Fx_{k-1} + Qv_{k-1}$$
state

$$z_k = Hx_k + Rw_k$$
measurement

Strict assumptions: linear models, Gaussian noise



$$\widehat{x}_{k} = F x_{k-1}$$

$$\widehat{P}_{k} = Q + F P_{k-1} F^{T}$$

$$K = \widehat{P}_{k} H^{T} (H \widehat{P}_{k} H^{T} + R)^{-1}$$

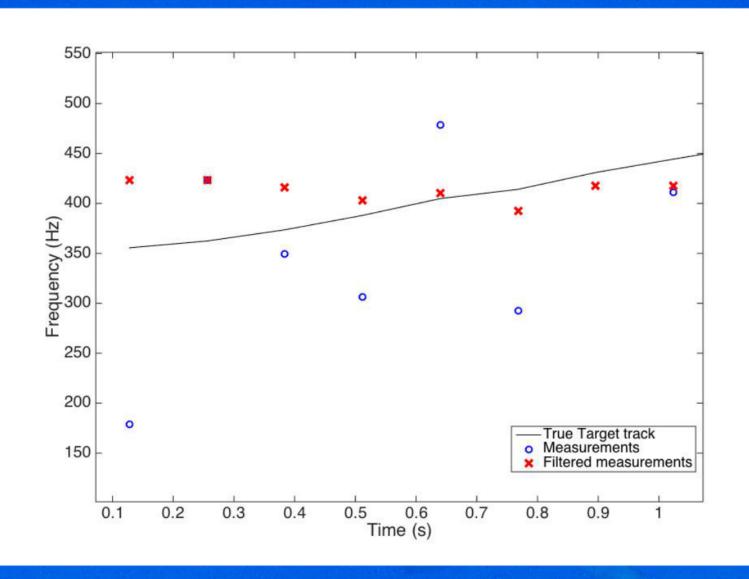
$$x_{k} = \widehat{x}_{k} + K (\mathbf{z} - H \widehat{x}_{k})$$

$$P_{k} = (I - KH) \widehat{P}_{k}$$

Predict state estimate

Kalman gain

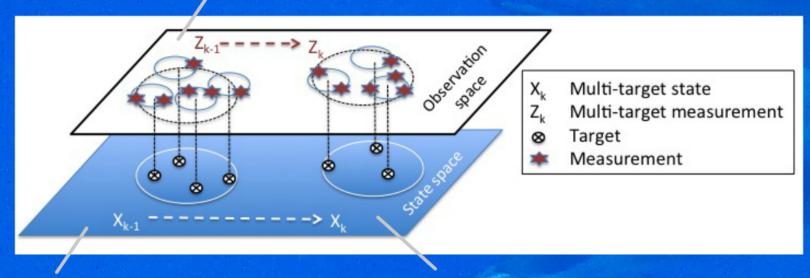
Update state estimate (refined)



Multi-target tracking

Goal: jointly estimate the number of targets and their states from noisy measurements.

measurements (targets + clutter)



multiple targets / no targets

number of targets change

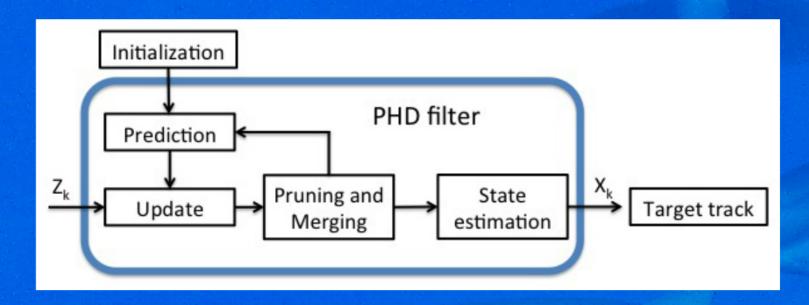
Traditional approaches: NN, GNN, JPDA, MHT; others: FISST (PHD filters)

Probability Hypothesis Density (PHD) filters

Based on Finite Set Statistics (FISST)

multi-source, multi-target problem single-source, single-target problem

 Propagates first-order statistical moment (intensity function/PHD) – strategy analogous to Kalman filter



Gaussian Mixture PHD filter [1]

- Approximates PHD filter
- Recursive algorithm, propagates intensity function or PHD that is represented by a sum of weighted Gaussian components

Each target ≈ **Gaussian component**

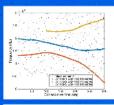
Means and covariances predicted & updated with Kalman filter equations

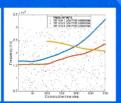
Weights predicted & updated with PHD equation

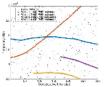
Assumptions: linear Gaussian system and measurement models

GM-PHD filter applications

Synthetic data



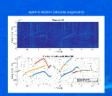


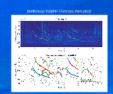


50 data sets Precision: 98.6 (±5.6) Recall: 98 (±8.5) Fragments: 1.2 (±0.2) Coverage: 96.6 (±9.4)

Real data

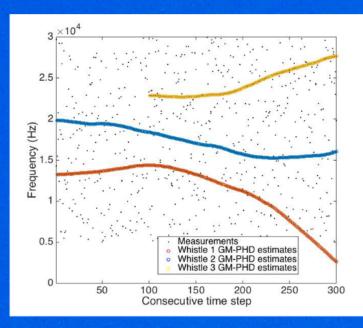
(data taken from MobySound.org)

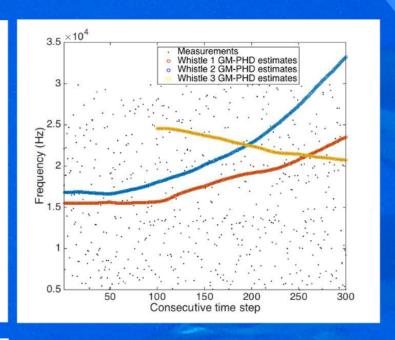


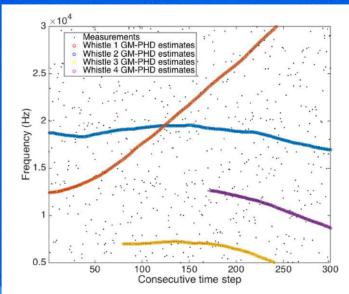




Synthetic data





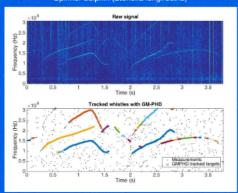


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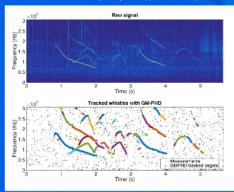
Real data

(data taken from MobySound.org)

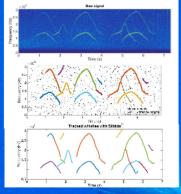
Spinner dolphin (Stenella longirostris)



Bottlenose dolphin (Tursiops truncatus)



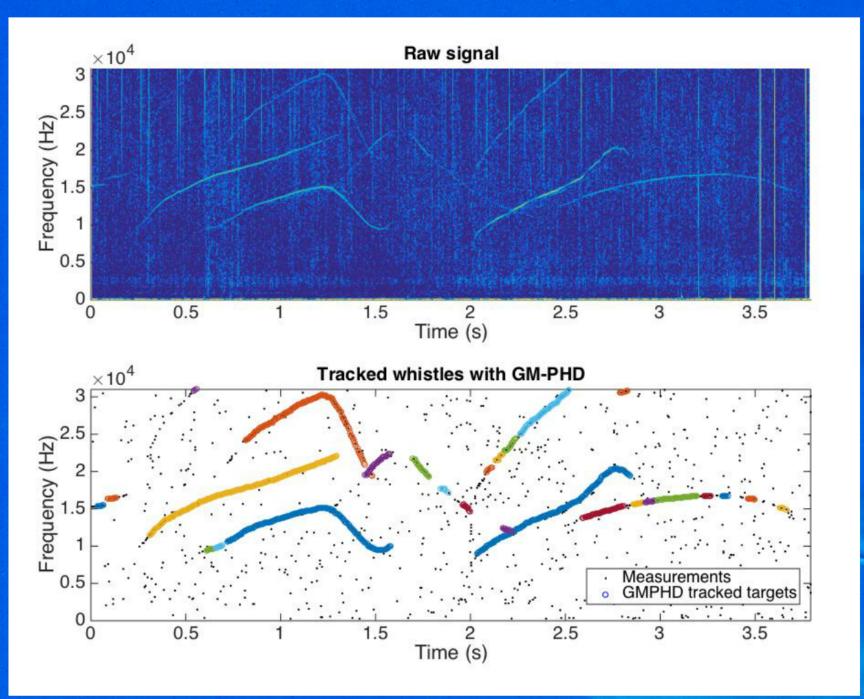
Short-beaked common dolphin (Delphinus delphis)



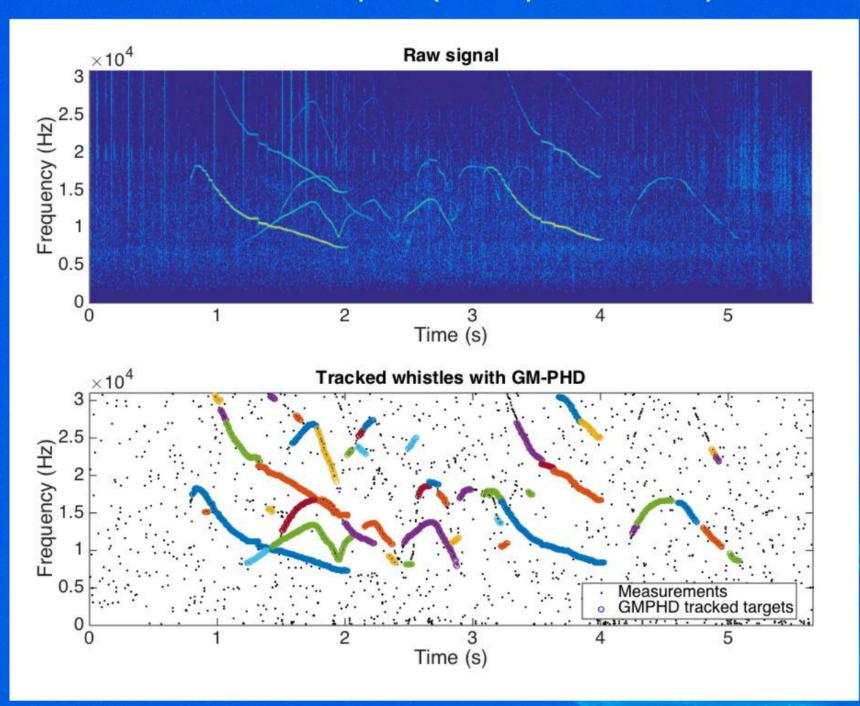
- Threshold 10 dB; - Window length 10.7 ms (50% overlap); - Minimum whistle length 150 ms

[2] Roch et al. JASA (2011)

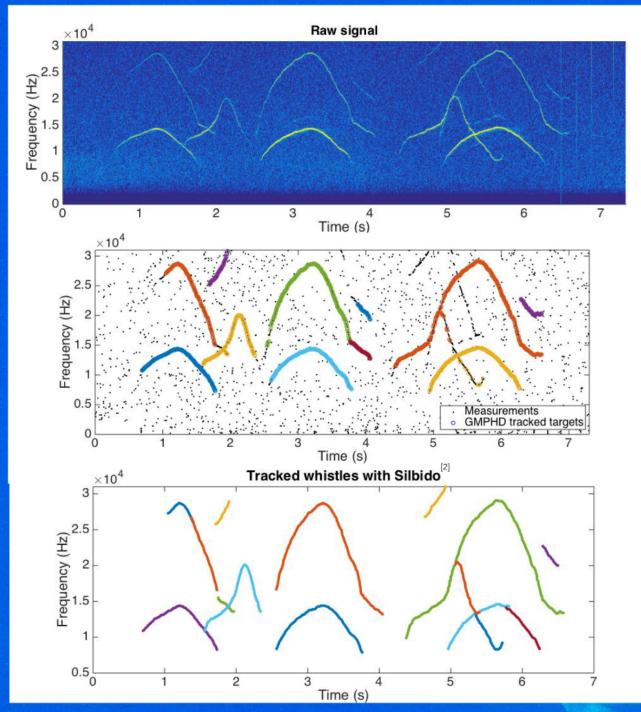
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Challenges

PSEUDO-CODE FOR THE GAUSSIAN MIXTURE PHD FILTER

- Optimization of GM-PHD filter parameters
- Models (system, measurement, birth and clutter)
- Assumes linear models

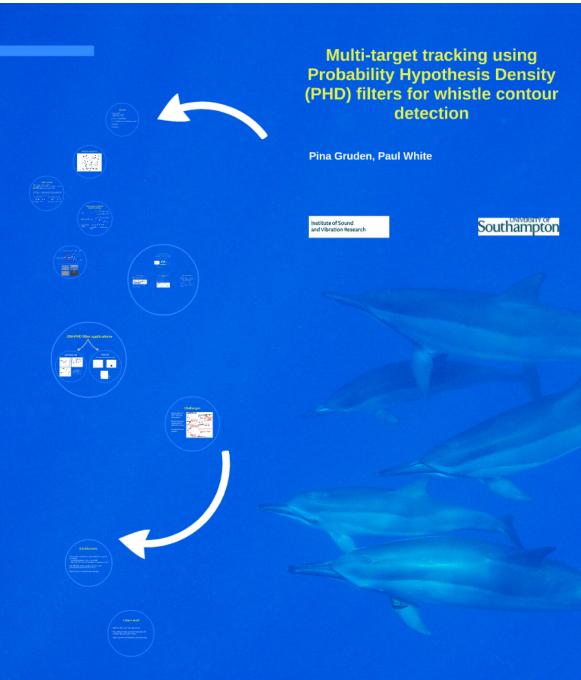
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PRUNING FOR THE GAUSSIAN MIXTURE PHD FILTER.
given \{w_{k-1}^{(i)}, m_{k-1}^{(i)}, P_{k-1}^{(i)}\}_{i=1}^{J_{k-1}}, and the measurement set Z_k
step 1. (prediction for birth targets)
                                                                                             given \{w_k^{(i)}, m_k^{(i)}, P_k^{(i)}\}_{i=1}^{J_k}, a truncation threshold T, a merging thresh-
   i = 0.
                                                                                             old U, and a maximum allowable number of Gaussian terms J_{max}
   for j = 1, \dots, J_{\gamma,k}
                                                                                             Set \ell = 0, and I = \{i = 1, ..., J_k | w_k^{(i)} > T\}.
       i := i + 1.
        w_{k|k-1}^{(i)} = w_{\gamma,k}^{(j)}, \quad m_{k|k-1}^{(i)} = m_{\gamma,k}^{(j)}, \quad P_{k|k-1}^{(i)} = P_{\gamma,k}^{(j)}
                                                                                                                                                    Pruning threshold
   for j = 1, \dots, J_{\beta,k}
                                                                                                  j := \arg \max_{k} w_k^{(i)}.
        for \ell = 1, ..., J_{k-1}
            i := i + 1.
                                                                                                   L := \left\{ i \in I \mid (m_k^{(i)} - m_k^{(j)})^T (P_k^{(i)})^{-1} (m_k^{(i)} - m_k^{(j)}) \right\} 
            w_{k|k-1}^{(i)} = w_{k-1}^{(\ell)} w_{\beta,k}^{(j)}
            m_{k|k-1}^{(i)} = d_{\beta,k-1}^{(j)}
                                      Probability of
                                                                                                   	ilde{m}_k^{(\ell)} = rac{i \in L}{a_n^{(\ell)}} \sum_{w_k^{(i)} x_k^{(i)}} 	ext{Merging threshold}
                                      target survival
                                                                                                  \tilde{P}_k^{(\ell)} = \frac{1}{\tilde{w}_k^{(\ell)}} \sum_{i=1}^{k} w_k^{(i)} (P_k^{(i)} + (\tilde{m}_k^{(\ell)} - m_k^{(i)}) (\tilde{m}_k^{(\ell)} - m_k^{(i)})^T).
         \begin{array}{l} w_{k|k-1}^{(i)} = p_{S,k} w_{k-1}^{(j)}, \\ m_{k|k-1}^{(i)} = F_{k-1} m_{k-1}^{(j)}, \quad P_{k|k-1}^{(i)} = Q_{k-1} + F_{k-1} P_{k-1}^{(j)} F_{k-1}^T, \end{array} 
                                                                                            if \ell > J_{max} then replace \{\tilde{w}_k^{(i)}, \tilde{m}_k^{(i)}, \tilde{P}_k^{(i)}\}_{i=1}^{\ell} by those of the J_{max}
                                                                                                    sians with largest weights.
                                                                                                    ut \{\tilde{w}_k^{(i)}, \tilde{m}_k^{(i)}, \tilde{P}_k^{(i)}\}_{i=1}^{\ell} as pruned Gaussian components.
   for j=1,\dots,J_{k|k-1}
                                         Probability of
        \eta_{k|k-1}^{(j)} = H_k m_{k|k-1}^{(j)}
                                        target detection
                                                                                                                                              TABLE III
                                                                                                                         MULTI-TARGET STATE EXTRACTION
        w_{k}^{(j)} = (1 \cdot (p_{D,k}) w_{k|k-1}^{(j)},
        m_k^{(j)} = m_{k|k-1}^{(j)}, P_k^{(j)} = P_{k|k-1}^{(j)}.
                                                                                            given \{w_k^{(i)}, m_k^{(i)}, P_k^{(i)}\}_{i=1}^{J_k}.
   for each z \in Z_k
                                                                                            Set \hat{X}_k = \emptyset.
        \ell := \ell + 1.
                                                                                             for i = 1, \dots, J_k
         for j = 1, ..., J_{k|k-1}
                                                                                                                                         Weight threshold
              w_{k|k-1}^{(\ell J_{k|k-1}+j)} = p_{D,k} w_{k|k-1}^{(j)} \mathcal{N}(z; \eta_{k|k-1}^{(j)}, S_k^{(j)})
              m_k^{(\ell J_{k|k-1}+j)} = m_{k|k-1}^{(j)} + K_k^{(j)}(z - \eta_{k|k-1}^{(j)}),
                                                                                                        for j = 1, \ldots, \text{round}(w_k^{(1)})
                                                                                                             update \hat{X}_k := \left[\hat{X}_k, m_k^{(i)}\right]
                                                                                                  end
         1, \dots, J_{k|k-1}.
    J_k = \ell J_{k|k-1} + J_{k|k-1}.
                                       Clutter intensity utput \hat{X}_k as the multi-target state estimate.
output \{w_k^{(i)}, m_k^{(i)}, P_k^{(i)}\}_{i=1}^{J_k}.
```

Conclusions

- Successfully implemented automated MTT using GM-PHD filters
 - Synthetic problems (works very well)
 - Real data (occasional "breaking" of contours occurs)
- GM-PHD filter shows considerable promise to simultaneously track whistle contours
- Filter fast and computationally reasonable

Future work

- Optimize filter and model parameters
- Run extensive tests on real whistle data and evaluate detector performance
- Implement SMC-PHD filter for whistle tracking



Thank you!

Any Questions?