Acoustics Lab #8 and #9
Reflection and Transmission

Define velocity potential \( \vec{u} = -\nabla \psi \)

Newton's law \( \left( \frac{\partial \vec{u}}{\partial t} \right) = -\nabla P \)

Continuity \( K \frac{\partial^2 \psi}{\partial t^2} = -\nabla \cdot \vec{u} \)

\( K \) = compressibility

Use potential
\( \rho \frac{\partial}{\partial t} \left( -\nabla \psi \right) = -\nabla P \)
\( \rho \frac{\partial \psi}{\partial t} = P \)

And
\( K \frac{\partial^2 \psi}{\partial t^2} = +\nabla \cdot \nabla \psi = \nabla^2 \psi \)

\( K \rho \frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi \) WAVE EQUATION FOR POTENTIAL

For plane wave: Pressure is in phase with fluid velocity:
\( P_x = \rho c u_x = -\rho c \psi_x \)

Greatest pressure is in region moving the fastest.
\( \frac{P}{u} = \rho c \) is called the "characteristic impedance"

\( 1.5 \times 10^6 \) MKS for water
Reflection from locally reactive surface:
Assume: parts of reflecting surface are not strongly coupled

In order to match \( \psi_i \) and \( \psi_r \) at the boundary \( y=0 \) requires \( \theta_i = \theta_r \)

\[
\psi = -j \epsilon c \left[ \psi_i + \psi_r \right] = -j \epsilon c \left[ 1 + cr \right] \psi_i
\]

Let \( \psi_r = cr \psi_i \) call \( cr \) the reflection coefficient

Define impedance of boundary:

\[
Z = \frac{P}{-uy}
\]

at boundary:

\[
uy(x,0) = \cos \theta \psi_i - \cos \theta cr \psi_i
\]

incident \hspace{1cm} reflected

\[
Z \frac{\cos \theta}{\epsilon c} = \frac{1 + cr}{1 - cr}
\]

\[
\frac{Z \cos \theta}{\epsilon c} - 1 = cr \left( \frac{Z \cos \theta}{\epsilon c} + 1 \right)
\]

\[
(1 - cr) = \left( \frac{Z \cos \theta - \epsilon c}{Z \cos \theta + \epsilon c} \right)
\]
Assume: \( Z > R \) and \( Z = R \) is real

Then at \( \theta = \cos^{-1} \left( \frac{R}{Z} \right) \) \(|Cr|^2 = 0\).

For \( Z = iX \) imaginary

\[
|Cr|^2 = \left( \frac{iX \cos \theta - Ec}{iX \cos \theta + Ec} \right) \left( \frac{-iX \cos \theta - Ec}{-iX \cos \theta + Ec} \right) = 1
\]